

# QCD Sum Rule Analysis for the $\Lambda_b \rightarrow \Lambda_c$ Semileptonic Decay

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## Abstract

The  $1/m_c$  and  $1/m_b$  corrections to the  $\Lambda_b \rightarrow \Lambda_c$  semileptonic decay are analyzed by QCD sum rules. Within the framework of heavy quark effective theory, the subleading baryonic Isgur-Wise function of  $\Lambda_b \rightarrow \Lambda_c$  has been calculated. It is shown that the corrections due to the  $1/m$  Lagrangian insertion are negligibly small. The sizable  $1/m_Q$  effect to the decay lies only in the weak current. The decay spectrum and the branching ratio are given.

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The weak decays of heavy baryons provide testing ground for the Standard Model. They reveal some important features of the physics of heavy quarks. From the study of the heavy quark physics, some important parameters of the Standard Model, for instance, the Cabbibo-Kobayashi-Maskawa (CKM) matrix element  $V_{cb}$  can be extracted by comparing experiments with theoretical calculations from the decay mode  $\Lambda_b \rightarrow \Lambda_c l \bar{\nu}$ .

The main difficulties in the Standard Model calculations are due to the poor understanding of the nonperturbative aspects of the strong interactions (QCD). Besides the numerical lattice methods, some analytic, model-independent nonperturbative QCD methods have been developed. For the heavy hadrons containing a single heavy quark, an effective theory of QCD based on the heavy quark symmetry in the heavy quark limit [1], the so-called heavy quark effective theory (HQET), has been proposed [2]. The classification of the weak decay form factors of heavy baryons has been simplified greatly in HQET [3]. To increase the precision of the analysis, subleading corrections [4] to the results in the heavy quark limit have also been considered for baryons [5]. However, for a complete analysis to the heavy baryons, we still need to employ some other nonperturbative methods.

Combining the QCD sum rule [6] method, the complete analysis for heavy baryons can be made in HQET. As a nonperturbative method rooted in QCD itself, QCD sum rule has been applied successfully to calculate the properties of various hadrons [6, 7]. For the heavy mesons, it has been used in the framework of HQET to the leading order heavy quark expansion to calculate the masses, the decay constants and the Isgur-Wise function [8]. And  $1/m_Q$  corrections have also been calculated [9, 10]. Heavy baryons were first calculated by QCD sum rules in Ref. [11]. The heavy baryon masses and the baryonic Isgur-Wise functions have been calculated in the HQET sum rules to the leading order heavy quark expansion in Refs. [12] and [13] respectively. We [14] and another group [15] have calculated the  $1/m_Q$  corrections to heavy baryon masses of

the results of Ref. [12]. In this paper, the subleading Isgur-Wise function of the weak transition  $\Lambda_b \rightarrow \Lambda_c$  is further studied in the HQET sum rules.

The hadronic matrix element of the weak current for  $\Lambda_b \rightarrow \Lambda_c$  is parameterized generally by six form factors  $F_i$  and  $G_i$  ( $i = 1, 2, 3$ ),

$$\begin{aligned} \langle \Lambda_c(v') | \bar{c} \gamma^\mu (1 - \gamma^5) b | \Lambda_b(v) \rangle = & \bar{u}_{\Lambda_c}(v') (F_1 \gamma^\mu + F_2 v^\mu + F_3 v'^\mu) u_{\Lambda_b}(v) \\ & - \bar{u}_{\Lambda_c}(v') (G_1 \gamma^\mu + G_2 v^\mu + G_3 v'^\mu) \gamma^5 u_{\Lambda_b}(v) , \end{aligned} \quad (1)$$

where  $v$  and  $v'$  denote the four-velocities of  $\Lambda_b$  and  $\Lambda_c$  respectively. These form factors need to be determined by some nonperturbative QCD method. Within the framework of HQET, the classification of them is simplified very much. To the order of  $1/m_Q$ , the effective Lagrangian for the heavy quark  $h_v$  is

$$\begin{aligned} \mathcal{L}_{\text{eff}} &= \bar{h}_v i v \cdot D h_v + \frac{1}{2m_Q} \mathcal{L}' \\ \mathcal{L}' &= \bar{h}_v (iD)^2 h_v - \frac{g}{2} \bar{h}_v \sigma_{\mu\nu} G^{\mu\nu} h_v . \end{aligned} \quad (2)$$

In the heavy quark limit, the form factors are determined by only one independent function  $\xi(y)$ ,

$$\langle \Lambda_c(v') | \bar{h}_{v'}^{(c)} \Gamma h_v^{(b)} | \Lambda_b(v) \rangle = \xi(y) \bar{u}_{\Lambda_c}(v') \Gamma u_{\Lambda_b}(v) , \quad (3)$$

where  $y = v \cdot v'$  and  $\Gamma$  is some gamma matrix. To the order of  $1/m_Q$ , they are determined by one mass parameter  $\bar{\Lambda}$  and one additional function  $\chi(y)$  which are defined as follows,

$$\bar{\Lambda} = m_{\Lambda_Q} - m_Q , \quad (4)$$

and

$$\langle \Lambda_c(v') | \Gamma \bar{h}_{v'}^{(c)} \Gamma h_v^{(b)} i \int d^4x \frac{\mathcal{L}'(x)}{2m_Q} | \Lambda_b(v) \rangle = \frac{\bar{\Lambda}}{m_Q} \chi(y) \bar{u}_{\Lambda_c}(v') \Gamma u_{\Lambda_b}(v) . \quad (5)$$

Both the leading order universal function  $\xi$  and the subleading one  $\chi$  are called Isgur-Wise function. While  $\xi$  and  $\bar{\Lambda}$  have been calculated by the QCD sum rules, we are going to calculate the subleading Isgur-Wise function  $\chi$ .

QCD sum rule is a calculation method for some nonperturbative physical quantities [8]. The Green's function, from which the Isgur-Wise function can be obtained, is the three-point correlator of the heavy baryonic currents  $\tilde{j}$ 's and the weak current in HQET. Generally the current of the heavy  $\Lambda$ -baryons is

$$\tilde{j}^v = \epsilon^{abc}(q_1^T C \tilde{\Gamma} \tau q_2^b) h_v^c, \quad (6)$$

where  $C$  is the charge conjugate matrix,  $\tau$  is an antisymmetric flavor matrix,  $a, b, c$  denote the color indices, and the choice of the gamma matrix  $\tilde{\Gamma}$  is not unique, there are two choices,

$$\tilde{\Gamma}_1 = \gamma_5 \quad \text{and} \quad \tilde{\Gamma}_2 = \not{x} \gamma_5. \quad (7)$$

The current (6) is denoted as  $\tilde{j}_1^v$  for  $\tilde{\Gamma}_1$  and  $\tilde{j}_2^v$  for  $\tilde{\Gamma}_2$  respectively in the following. Before performing the sum rule analysis for the three-point correlator, which is required to obtain the subleading function  $\chi$ , let us first review some of the two-point correlator results of QCD sum rule [14], because they are related to the three-point correlator analysis.

In Ref. [14], we obtained the heavy baryon masses and the so-called baryonic "decay constants" to the order of  $1/m_Q$  by the QCD sum rule analysis of some two-point correlators. With the definition of the "decay constant"  $f$  in HQET

$$\langle 0 | \tilde{j}^v | \Lambda_Q \rangle = f_\Lambda u, \quad (8)$$

where  $u$  is the spinor in HQET, the sum rule gives <sup>2</sup>

$$\begin{aligned} 8f_{\Lambda_1}^2 e^{-2\bar{\Lambda}_1/T} &= \frac{1}{5 \times 2^5 \pi^4} \int_0^{\omega_c} d\omega \omega^5 e^{-\omega/T} + \frac{4}{3} \langle \bar{q}q \rangle^2 e^{-\frac{m_0^2}{2T^2}} + \frac{\langle \alpha_s GG \rangle}{2^4 \pi^3} T^2 \\ &\quad - \frac{1}{m_Q} \left( \frac{3}{5 \times 2^7 \pi^4} \int_0^{\omega_c} d\omega \omega^6 e^{-\omega/T} + \frac{m_0^2 \langle \bar{q}q \rangle^2}{T} e^{-\frac{m_0^2}{2T^2}} + \frac{13 \langle \alpha_s GG \rangle}{3 \times 2^5 \pi^3} T^3 \right), \end{aligned} \quad (9)$$

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<sup>2</sup>There are some errors in the coefficients of the gluon condensates in the  $1/m_Q$  corrections in Ref. [14]. For  $\Lambda$  baryons, the coefficients are modified in this paper. Besides, in Eq. (20) of Ref. [14], the coefficients  $\frac{13}{3}$  and  $\frac{5}{3}$  should be replaced by 3. However these modifications do not affect the numerical results of Ref. [14].

for  $\tilde{j}_1^v$ , and

$$8f_{\Lambda_2}^2 e^{-2\bar{\Lambda}_2/T} = \frac{1}{5 \times 2^5 \pi^4} \int_0^{\omega_c} d\omega \omega^5 e^{-\omega/T} + \frac{4}{3} \langle \bar{q}q \rangle^2 e^{-\frac{m_0^2}{2T^2}} + \frac{\langle \alpha_s GG \rangle}{2^4 \pi^3} T^2 - \frac{1}{m_Q} \left( \frac{1}{2^7 \pi^4} \int_0^{\omega_c} d\omega \omega^6 e^{-\omega/T} + \frac{m_0^2 \langle \bar{q}q \rangle^2}{T} e^{-\frac{m_0^2}{2T^2}} + \frac{19 \langle \alpha_s GG \rangle}{3 \times 2^5 \pi^3} T^3 \right), \quad (10)$$

for  $\tilde{j}_2^v$ . In above equations,  $T$  is the Borel parameter. And  $\omega_c$  is the continuum threshold.

The three-point correlator  $\tilde{\Xi}(\omega, \omega', y)$  which we choose for sum rule analysis in the HQET is

$$\tilde{\Xi}_{ij}(\omega, \omega', y) = i^2 \int d^4x' d^4x e^{ik'x' - ikx} \langle 0 | T \tilde{j}_i^{v'}(x') \bar{h}_{v'}^{(Q')}(0) \Gamma h_v^{(Q)}(0) \tilde{j}_j^v(x) | 0 \rangle, \quad i, j = 1, 2, \quad (11)$$

where  $\omega = 2v \cdot k$  and  $\omega' = 2v' \cdot k'$ . Because of the heavy quark symmetry,  $m_Q$  and  $m_{Q'}$  are taken to be equal for simplicity. The hadronic representation of this correlator is

$$\tilde{\Xi}_{ij}(\omega, \omega', y) = \left[ \frac{4f^2(\xi + \frac{2\bar{\Lambda}}{m_Q}\chi)}{(2\bar{\Lambda} - \omega)(2\bar{\Lambda} - \omega')} \right]_{ij} \frac{1 + \not{v}'}{2} \Gamma \frac{1 + \not{v}}{2} + \text{res.}, \quad (12)$$

where  $\bar{\Lambda}$  and  $f^2$  have been given in the sum rules (9) and (10) to the order of  $1/m_Q$ . On the other hand,  $\tilde{\Xi}(\omega, \omega', y)$  can be calculated in terms of quark and gluon language with vacuum condensates. This will establish the sum rule. Only the diagonal correlators ( $i = j$ ) will be considered. It should be remarked here that in general we can consider the correlation function of the linear combination  $\tilde{j}_1^v + \tilde{x}\tilde{j}_2^v$  with  $\tilde{x}$  being the mixing parameter. But with the commonly adopted quark-hadron duality, the mixed correlator  $\tilde{\Xi}_{12}$  has no perturbation term in the sum rule. Therefore, the effect due to the mixing is expected to be small.

The calculation of  $\tilde{\Xi}(\omega, \omega', y)$  are straightforward. In addition to the Feynman diagrams at the leading order heavy quark expansion which were given in Ref. [13], the diagrams of the  $1/m_Q$  corrections to the three-point correlator  $\tilde{\Xi}(\omega, \omega', y)$  are shown

in Fig. 1. They are calculated by including insertions of the  $1/m_Q$  operators of the Lagrangian (2) with standard method. The chromo-magnetic operator insertion is vanishing for the  $\Lambda_Q \rightarrow \Lambda_{Q'}$  transition. Therefore only the kinetic energy term insertions need to be considered in our case. Instead of the momentum representation, we adopt the coordinate representation in our calculation. The heavy quark propagator is in a very simple form in the coordinate representation so that the calculations become comparatively easy. Taking the insertion of the purely kinetic energy term at the order of  $1/m_Q$  into account, the heavy quark propagator is

$$\langle 0 | T h_v(x) \bar{h}_v(0) | 0 \rangle = \int_0^\infty dt (1 - i \frac{t}{2m_Q} \partial^\mu \partial_\mu) \delta(x - tv) \frac{1 + \not{v}}{2}. \quad (13)$$

The fixed point gauge [16] is used. All the condensates with dimensions lower than 6 are retained. We also include the dimension 6 condensate  $\langle \bar{q}(x)q(x') \rangle^2$  in our analysis which is a main contribution. We use the gaussian ansatz for the distribution in spacetime for this condensate [17]. We use the following values of the condensates,

$$\begin{aligned} \langle \bar{q}q \rangle &\simeq -(0.23 \text{ GeV})^3, \\ \langle \alpha_s GG \rangle &\simeq 0.04 \text{ GeV}^4, \\ \langle g \bar{q} \sigma_{\mu\nu} G^{\mu\nu} q \rangle &\equiv m_0^2 \langle \bar{q}q \rangle, \quad m_0^2 \simeq 0.8 \text{ GeV}^2. \end{aligned} \quad (14)$$

The normalization  $\text{Tr} \tau^\dagger \tau = 1$  has been used in the analysis. In the fixed-point gauge, the space-time translational invariance is violated, but it is restored by adding all the diagrams in Fig. 1. This is a check of our calculation.

We use the commonly adopted quark-hadron duality for the resonance part of Eq. (12). Generally the duality is to simulate the resonance contribution by the perturbative part above some threshold energy  $\omega_c$ . The perturbative contribution of the three-point correlator  $\tilde{\Xi}^{\text{pert}}(\omega, \omega', y)$  can be expressed by the dispersion relation,

$$\tilde{\Xi}^{\text{pert}}(\omega, \omega', y) = \frac{1}{\pi} \int_0^\infty d\tilde{\omega} \int_0^\infty d\tilde{\omega}' \frac{\text{Im} \tilde{\Xi}^{\text{pert}}(\tilde{\omega}, \tilde{\omega}', y)}{(\tilde{\omega} - \omega)(\tilde{\omega}' - \omega')}. \quad (15)$$

The integration domain is a kitelike area. With the redefinition of the integral variables,

$$\begin{aligned}\tilde{\omega}_+ &= \frac{\tilde{\omega} + \tilde{\omega}'}{2}, \\ \tilde{\omega}_- &= \left(\frac{y+1}{y-1}\right)^{1/2} \frac{\tilde{\omega} - \tilde{\omega}'}{2},\end{aligned}\tag{16}$$

the integration becomes

$$\int_0^\infty d\tilde{\omega} \int_0^\infty d\tilde{\omega}' \dots = 2\left(\frac{y-1}{y+1}\right)^{1/2} \int_0^\infty d\tilde{\omega}_+ \int_{-\omega_+}^{\omega_+} d\tilde{\omega}_- \dots \tag{17}$$

It is in  $\omega_+$ , that the quark-hadron duality is assumed [18],

$$\text{res.} = \frac{2}{\pi} \left(\frac{y-1}{y+1}\right)^{1/2} \int_{\omega_c}^\infty d\tilde{\omega}_+ \int_{-\omega_+}^{\omega_+} d\tilde{\omega}_- \frac{\text{Im}\tilde{\Xi}^{\text{pert}}(\tilde{\omega}, \tilde{\omega}', y)}{(\tilde{\omega} - \omega)(\tilde{\omega}' - \omega')}. \tag{18}$$

In the heavy quark limit, we have double checked the analysis of Ref. [13]. There are two sum rules for the leading order Isgur-Wise function corresponding to two choices of the baryonic current. When  $\omega_c$  lies between 1.8 – 2.5 GeV, the stability window of  $T$  exists,  $T = 0.3 - 0.6$  GeV. The two results for the Isgur-Wise function are consistent with each other. For  $y$  lies in the physical region 1 – 1.43, the linear approximation can fit the results,

$$\xi(y) = 1 - \rho(y-1), \quad \rho = 0.55 \pm 0.15, \tag{19}$$

where the uncertainty of  $\rho$  accounts those of  $\omega_c$  and  $T$ , in addition to the difference of the two sum rule results. For  $y$  lies in 1 – 3, we find that the following function fit very well to our numerical results for the Isgur-Wise function for reasonable  $\omega_c$  and  $T$ ,

$$\xi(y) = \left(\frac{2}{y+1}\right)^{0.5} \exp(-0.8 \frac{y-1}{y+1}). \tag{20}$$

We note that the  $y$ -dependence of the Isgur-Wise function is not as steep as that of the Skyrme model [19] and the quark model [20].

The sum rule for the subleading Isgur-Wise function  $\chi(y)$  is

$$\chi(y) = -\frac{e^{2\bar{\Lambda}/T}}{8\bar{\Lambda}f^2} [J(y) - \xi(y)J(1)], \tag{21}$$

where

$$\begin{aligned}
J_1(y) &= \left(\frac{1}{2\pi} \frac{1}{y+1}\right)^4 \frac{4y-1}{5} \int_0^{\omega_c} d\omega \omega^6 e^{-\omega/T} + \frac{m_0^2 \langle \bar{q}q \rangle^2}{6T} \\
&\quad \cdot \left[3 + \frac{m_0^2}{4T^2}(y^2 - 1)\right] e^{-\frac{m_0^2}{4T^2}(y+1)} + \frac{\langle \alpha_s GG \rangle}{3} \left(\frac{1}{2\pi} \frac{T}{y+1}\right)^3 (4y^2 + 3y + 6) , \\
J_2(y) &= \left(\frac{1}{2\pi} \frac{1}{y+1}\right)^4 y \int_0^{\omega_c} d\omega \omega^6 e^{-\omega/T} + \frac{m_0^2 \langle \bar{q}q \rangle^2}{6T} y \left[3 + \frac{m_0^2}{4T^2}(y^2 - 1)\right] e^{-\frac{m_0^2}{4T^2}(y+1)} \\
&\quad + \frac{\langle \alpha_s GG \rangle}{3} \left(\frac{1}{2\pi} \frac{T}{y+1}\right)^3 (2y^3 + 8y^2 + 4y + 5) ,
\end{aligned} \tag{22}$$

with the subscripts 1 and 2 denoting the two kinds of baryonic currents. The Luke's theorem [4] in the baryon case  $\chi(1) = 0$  is satisfied automatically. The numerical results are shown in Fig. 2 where the two curves correspond to the two sum rule results. The range of  $\omega_c$  is the same as that in the leading order. The sum rule window is narrower than the leading order one. In the window  $T = 0.35 - 0.55$  the results for the subleading Isgur-Wise function are stable. The two results can also be regarded as being consistent with each other. Nevertheless, it is obvious that the subleading Isgur-Wise function is negligibly small,

$$\chi(y) \simeq O(10^{-2}) . \tag{23}$$

The semileptonic decay  $\Lambda_b \rightarrow \Lambda_c l \bar{\nu}$  can be analyzed directly after obtaining the hadronic matrix elements from the QCD sum rules. By neglecting the lepton mass, it is easy to show that the differential decay rate is

$$\begin{aligned}
\frac{1}{\sqrt{y^2 - 1}} \frac{d\Gamma(\Lambda_b \rightarrow \Lambda_c l \bar{\nu})}{dy} &= \frac{G_F^2 |V_{cb}|^2 m_{\Lambda_b}^2 m_{\Lambda_c}^3}{(2\pi)^3} \{ (1 - 2ry + r^2) [(y-1)F_1^2 + (y+1)G_1^2] \\
&\quad + \frac{y^2 - 1}{3} (Ar^2 + 2Br + C) \} ,
\end{aligned} \tag{24}$$



where  $r = m_{\Lambda_c}/m_{\Lambda_b}$ . In the above equation,

$$\begin{aligned}
A &= 2F_1F_2 + (y+1)F_2^2 + 2G_1G_2 + (y-1)G_2^2, \\
B &= F_1^2 + F_1F_2 + F_2F_3 + F_3F_1 + yF_2F_3 + G_1^2 - G_1G_2 - G_2G_3 + G_3G_1 + yG_2G_3, \\
C &= (y+1)F_3^2 + 2F_1F_3 + (y-1)G_3^2 - 2G_1G_3.
\end{aligned} \tag{25}$$

To the order of both  $1/m_c$  and  $1/m_b$ , the form factors  $F_i$  and  $G_i$  are expressed as

$$\begin{aligned}
F_1 &= C(\mu)\xi(y) + \left(\frac{\bar{\Lambda}}{2m_c} + \frac{\bar{\Lambda}}{2m_b}\right)[2\chi(y) + \xi(y)], \\
G_1 &= C(\mu)\xi(y) + \left(\frac{\bar{\Lambda}}{2m_c} + \frac{\bar{\Lambda}}{2m_b}\right)[2\chi(y) + \frac{y-1}{y+1}\xi(y)], \\
F_2 &= G_2 = -\frac{\bar{\Lambda}}{m_c(y+1)}\xi(y), \\
F_3 &= -G_3 = -\frac{\bar{\Lambda}}{m_b(y+1)}\xi(y),
\end{aligned} \tag{26}$$

where  $C(\mu)$  is the perturbative QCD coefficient. The subleading Isgur-Wise function can be safely neglected. The  $1/m_Q$  corrections are mainly due to the weak current. With the form of the leading order Isgur-Wise function (19), the differential decay rate of  $\Lambda_b \rightarrow \Lambda_c l \bar{\nu}$  is shown in Fig. 3. In Fig. 3, we have taken the heavy quark masses  $m_b = 4.83$  GeV,  $m_c = 1.44$  GeV and  $\bar{\Lambda} = 0.79$  GeV [14], the renormalization point  $\mu = 470$  MeV, the CKM matrix element  $V_{cb} = 0.04$  [21]. The width and the branching ratio of this decay mode are

$$\begin{aligned}
\Gamma &= 6.05 \times 10^{-14} \text{ GeV}, \\
\text{Br} &= 9.8\%.
\end{aligned} \tag{27}$$

The  $1/m_Q$  correction possesses 10% in the above branching ratio.

We have analyzed the  $\Lambda_b \rightarrow \Lambda_c$  semileptonic decays by QCD sum rules within the framework of HQET to the order of  $1/m_c$  and  $1/m_b$ . In the heavy quark limit, the analysis for the  $\Lambda_b \rightarrow \Lambda_c$  decay depends on one independent form factor which is the leading order Isgur-Wise function and was calculated in the QCD sum rules [13]. However, for a more precise analysis, only leading order calculation is not enough.

In this paper, we have considered the  $1/m_Q$  corrections. The subleading Isgur-Wise function has been calculated by the HQET sum rules. It is shown to be so small that it can be neglected. The  $1/m_Q$  correction to the decay  $\Lambda_b \rightarrow \Lambda_c$  results only from the weak current. The decay differential distribution has been given. The branching ratio is predicted to be  $\text{Br}(\Lambda_b \rightarrow \Lambda_c l \bar{\nu}) = 9.8\%$  after taking  $V_{cb} = 0.04$ . This will be useful to the experiments in the near future. The polarization effects of this decay have not been calculated which will be considered elsewhere.

Finally we would like to make a remark on the perturbative QCD corrections in the sum rule calculations. Such corrections to the baryonic Isgur-Wise function which still have not been included, would involve us in the three-loop calculations. However, we expect that they should be small. The Isgur-Wise function obtained from the QCD sum rule actually is a ratio of the three-point correlator to the two-point correlator results. While both of these correlators subject to large perturbative QCD corrections, their ratio does not depend on these corrections significantly because of cancelation. Therefore the results for the Isgur-Wise function are more reliable than that for the heavy baryon masses. This is what happened in the heavy meson case [8]. The perturbative QCD corrections to the two-point correlators, therefore to the heavy baryon masses, will be calculated elsewhere.

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## Figure captions

Fig. 1. Feynman diagrams for the  $1/m_Q$  corrections to  $\tilde{\Xi}(\omega, \omega', y)$ . The insertions are only the kinetic energy terms at the order of  $1/m_Q$ .

Fig. 2. Subleading Isgur-Wise function  $\chi(y)$ . The lower and the upper curves correspond to the sum rules (29) of  $J = J_1$  with  $\omega_c = 2.2$  GeV,  $T = 0.55$  GeV and  $J_2$  with  $\omega_c = 2.5$  GeV,  $T = 0.39$  GeV respectively.

Fig. 3. The differential decay rate of  $\Lambda_b \rightarrow \Lambda_c l \bar{\nu}$ . ( $y = v \cdot v'$ )

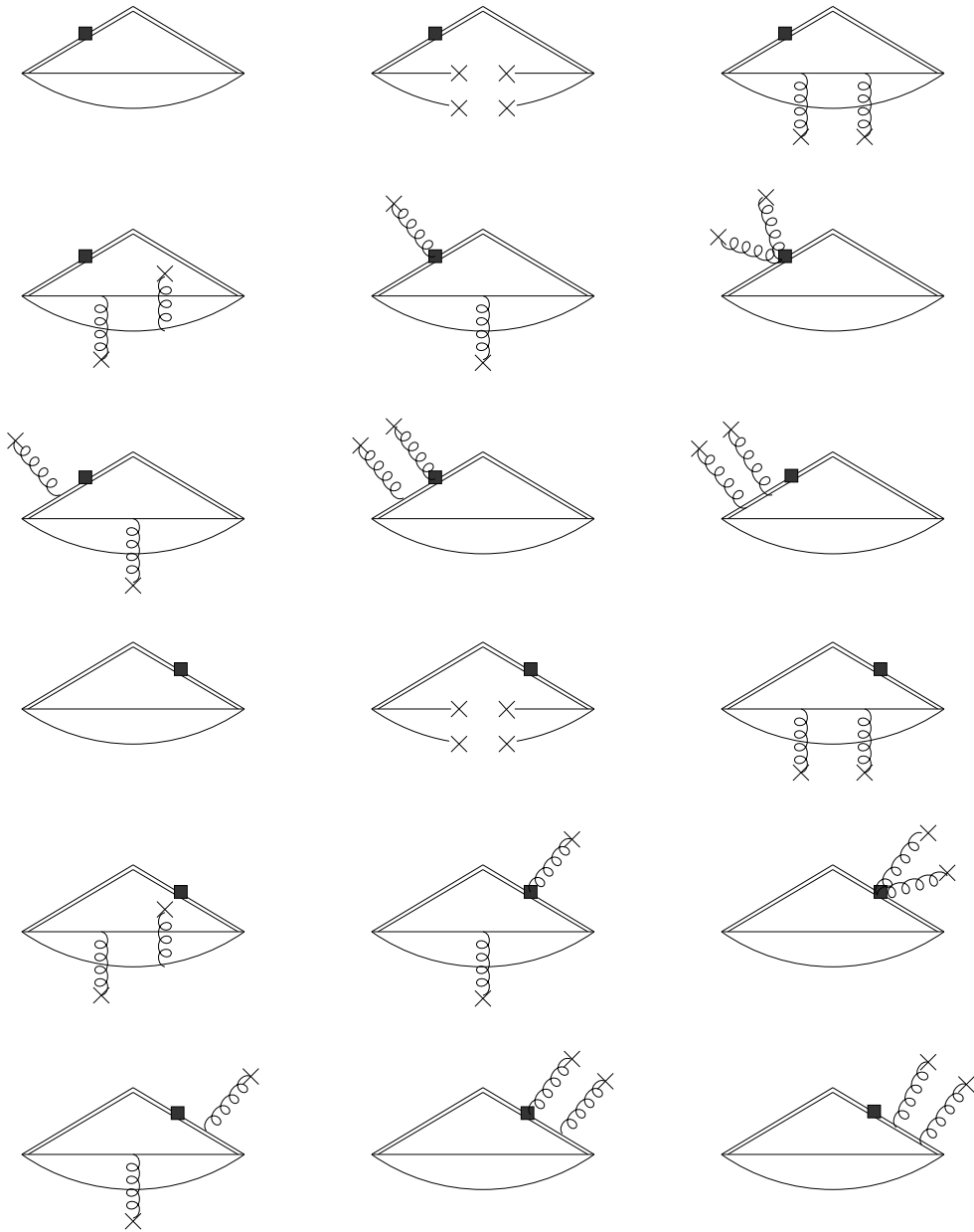
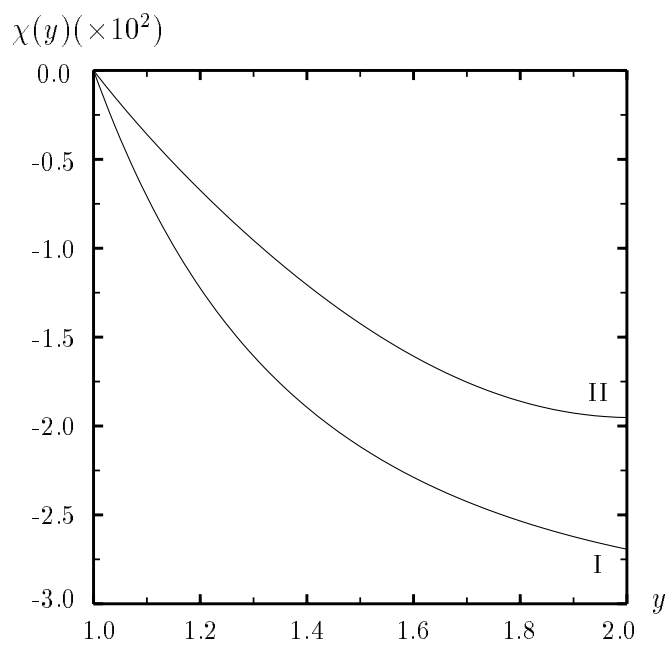
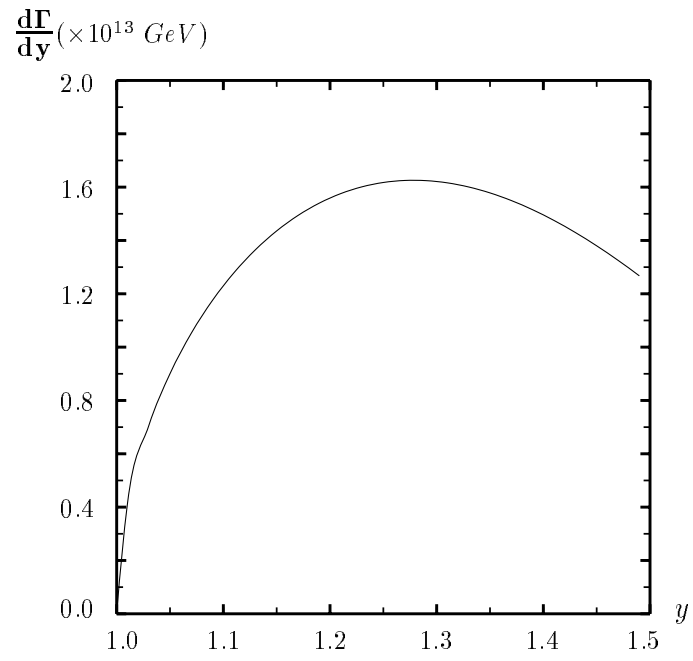


Fig. 1



**Fig 2**

**Fig 3**